

A Design Example

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The design starts from a 5th order normalized Chebychev lowpass filter with $\rho = 25\%$, which we augment with two zeros in the stopband at $\omega_{s1} = 1.3481$ and $\omega_{s2} = 1.9480$ rad/s (there is also one at infinity).

These are in fact the transmission zeros belonging to a 5th order Caer filter –C05 25 50 (Zverev, pgs. 220, 221)– with the same reflection coefficient.

Next to these zeros, also one Unit Element has been added according to the Sharpe method in a way that it contributes to the overall order of the circuit.

We can describe the approximation of the ideal low pass characteristic now with

$$|H(s)|_{s=j\omega}^2 = \frac{1}{1 + \epsilon^2 \cdot C \cdot R^2(\omega)}$$

in which R is a rational function, and with

$$\epsilon^2 = 10^{0.1r_p} - 1 = 0.06667$$

since it can be calculated that a $\rho = 25\%$ corresponds to a ripple in the passband of $r_p = 0.2803$ dB.

The multiplication constant C will be chosen such that $[C \cdot R^2(\omega)]_{\omega=1} = 1$.

We can derive (in our case using the method described by Vlach) that the rational function R used in the filter approximation will show 3 zeros in the passband, viz. at $\omega_{p1} = 0.28332$, $\omega_{p2} = 0.75083$ and at $\omega_{p3} = 0.97459$ rad/s.

Knowing the zeros and poles of R and taking the additional Unit Element into account, we can determine $R^2(\omega)$:

$$R^2(\omega) = \frac{\omega^{12} - 3.1877\omega^{10} + 3.8542\omega^8 - 2.1801\omega^6 + 0.5686\omega^4 - 0.0565\omega^2 + 0.0018}{\omega^{10} - 10.2242\omega^8 + 34.0640\omega^6 - 32.1180\omega^4 - 29.8459\omega^2 + 47.5603}$$

In order that $[C \cdot R^2(\omega)]_{\omega=1} = 1$, we need a $C = 25744$.

Rewriting $|H(s)|_{s=j\omega}^2$ as a rational function of Belevitch polynomials, we find

$$|H(s)|_{s=j\omega}^2 = \frac{1}{1 + \frac{h h^*}{f f^*}} = \frac{f f^*}{f f^* + h h^*} = \frac{f f^*}{g g^*}$$

in which

$$f f^* = \omega^{10} - 10.2242\omega^8 + 34.0640\omega^6 - 32.1180\omega^4 - 29.8459\omega^2 + 47.5603$$

$$h h^* = 1716.27\omega^{12} - 5470.90\omega^{10} + 6614.84\omega^8 - 3741.63\omega^6 + 975.856\omega^4 - 96.9237\omega^2 + 3.1707$$

$$g g^* = 1716.27\omega^{12} - 5469.90\omega^{10} + 6604.62\omega^8 - 3707.56\omega^6 + 943.738\omega^4 - 126.7696\omega^2 + 50.7309$$

$$g g_* = 1716.27s^{12} + 5469.90s^{10} + 6604.62s^8 + 3707.56s^6 + 943.738s^4 + 126.7696s^2 + 50.7309$$

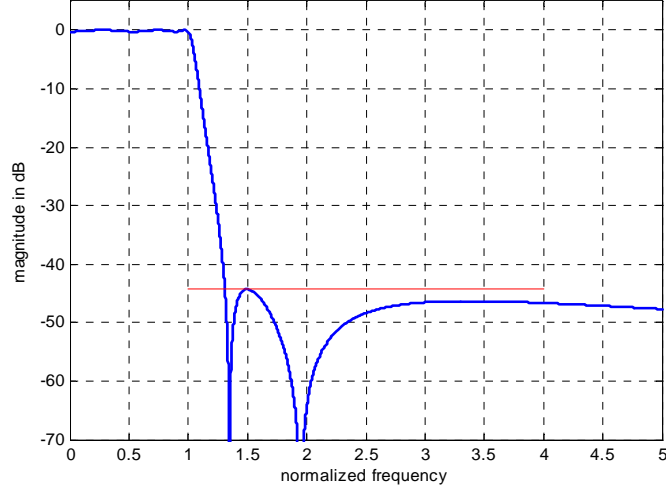


Figure 1: Amplitude Transfer Characteristic of the resulting low-pass filter in the w-plane

By determining the zeros of $g g_*$ and keeping only those in the left halfplane, we obtain $g(s)$:

$$g(s) = 41.4279s^6 + 55.6100s^5 + 103.3406s^4 + 85.9627s^3 + 66.2128s^2 + 28.5734s + 7.1226$$

$$h(s) = 41.4279s^6 + 66.0291s^4 + 27.2160s^2 + 1.7806$$

We can also calculate $f(s)$ and $h(s)$, which gives us a.o.

$$|H(s)| = \frac{f(s)}{g(s)} = \frac{s^5 - s^4 + 5.6121s^3 - 5.6121s^2 + 6.8964s - 6.8964}{41.4279s^6 + 55.6100s^5 + 103.3406s^4 + 85.9627s^3 + 66.2128s^2 + 28.5734s + 7.1226}$$

In Figure 1, the amplitude transfer characteristic $|H(s)|_{s=j\omega}$ in the w-plane is given.

Now, from the polynomials $g(s)$ and $h(s)$ we can derive the driving input impedance $Z_{in}(s)$ of the ladder structure to be

$$Z_{in}(s) = \frac{g(s) + h(s)}{g(s) - h(s)}, \text{ or}$$

$$Z_{in}(s) = \frac{82.8558s^6 + 55.6100s^5 + 169.3697s^4 + 85.9627s^3 + 93.4288s^2 + 28.5734s + 8.9032}{55.6100s^5 + 37.3115s^4 + 85.9627s^3 + 38.9969s^2 + 28.5734s + 5.3419}$$

We use this input impedance to synthesize the matching lossless twoport ladder network. From the impedance, we first extract an inductance, by assuming that this inductance is immediately followed by a series resonator circuit tuned to 1.9480 rad/s (ω_{s2}). The inductor's value can then be found from

$$L_s = \left[\frac{Z_{in}(s)}{s} \right]_{s=j\omega_{s2}}$$

and we obtain

$$Z_{in}(s) = 1.2749s + \frac{11.9559s^6 + 8.0398s^5 + 59.7717s^4 + 36.2437s^3 + 56.9991s^2 + 21.7627s + 8.9032}{55.6100s^5 + 37.3115s^4 + 85.9627s^3 + 38.9969s^2 + 28.5734s + 5.3419}$$

$$= L_1s + Z_1(s)$$

with $L_1 = 1.2749 \text{ H}$

Now we extract the resonator circuit from

$$Y_1(s) = \frac{1}{Z_1(s)} = \frac{55.6100s^5 + 37.3115s^4 + 85.9627s^3 + 38.9969s^2 + 28.5734s + 5.3419}{11.9559s^6 + 8.0398s^5 + 59.7717s^4 + 36.2437s^3 + 56.9991s^2 + 21.7627s + 8.9032}$$

and obtain

$$Y_1(s) = \frac{4.1979s}{s^2 + 1.9480^2} + \frac{5.4204s^3 + 3.5613s^2 + 4.9343s + 1.4077}{11.9559s^4 + 8.0398s^3 + 14.4024s^2 + 5.7350s + 2.3462}$$

or

$$Y_1(s) = \frac{\frac{1}{L_2}s}{s^2 + \frac{1}{L_2C_2}} + Y_2(s)$$

from which we find $L_2 = 0.2382 \text{ H}$ and $C_2 = 1.1062 \text{ F}$

Before the next element we want the Unit Element to appear. We calculate the Unit Element R_{UE1} using

$$Z_2(s) = \frac{1}{Y_2(s)} = \frac{11.9559s^4 + 8.0398s^3 + 14.4024s^2 + 5.7350s + 2.3462}{5.4204s^3 + 3.5613s^2 + 4.9343s + 1.4077}$$

by determining $R_{UE1} = [Z_2(s)]_{s=1}$, resulting in $R_{UE1} = 2.7721 \Omega$

We then derive a new $Z_3(s)$ from the previous $Z_2(s)$: if we define $Z_2(s) = \frac{b(s)}{a(s)}$, then $Z_3(s)$ can be written as

$$Z_3(s) = R_{UE1} \frac{R_{UE1} \cdot s \cdot a(s) - b(s)}{s \cdot b(s) - R_{UE1} \cdot a(s)}$$

$$Z_3(s) = \frac{8.5106s^4 + 5.0803s^3 - 2.0066s^2 - 5.0803s - 6.5040}{11.9559s^5 + 8.0398s^4 - 0.6236s^3 - 4.1374s^2 - 11.3323s - 3.9024}$$

Now both nominator and denominator can be divided by $s^2 - 1$, which reduces $Z_3(s)$ to

$$Z_3(s) = \frac{8.5106s^2 + 5.0803s + 6.5040}{11.9559s^3 + 8.0398s^2 + 11.3323s + 3.9024}$$

Now from the inverse $Y_3(s) = \frac{1}{Z_3(s)}$ it is possible to extract a capacitance, when we assume that this capacitance is followed by a resonator circuit tuned to 1.3481 rad/s (ω_{s1}).

We find C_3 from $C_3 = \left[\frac{Y_3(s)}{s} \right]_{s=j\omega_{s1}}$, and can rewrite $Y_3(s)$ as

$$Y_3(s) = 1.1599s + \frac{2.0845s^3 + 2.1473s^2 + 3.7884s + 3.9024}{8.5106s^2 + 5.0803s + 6.5040} = C_3 \cdot s + Y_4(s)$$

so $C_3 = 1.1599 \text{ F}$

From $Z_4(s) = \frac{1}{Y_4(s)} = \frac{8.5106s^2 + 5.0803s + 6.5040}{2.0845s^3 + 2.1473s^2 + 3.7884s + 3.9024}$

we can derive the elements of the parallel resonator, viz.

$$Z_4(s) = \frac{2.3659s}{s^2 + 1.3481^2} + \frac{3.5788}{2.0845s + 2.1473}$$

where

$$Z_4(s) = \frac{\frac{1}{C_4}s}{s^2 + \frac{1}{L_4C_4}} + Z_5(s)$$

which gives us $L_4 = 1.3018 \text{ H}$ and $C_4 = 0.4227 \text{ F}$

Finally from

$$Y_5(s) = \frac{1}{Z_5(s)} = \frac{2.0845s + 2.1473}{3.5788} = 0.5825s + 0.6000 = C_5s + \frac{1}{R_o}$$

we find the last two elements to be

$$C_5 = 0.5825 \text{ F}$$

$$R_o = 1.6667 \Omega$$

Figure 2 shows the resulting ladder structure.

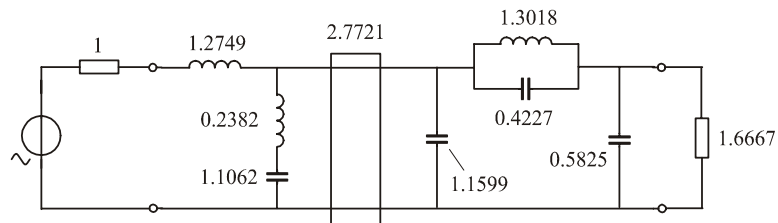


Figure 2: The topology and element values of the resulting ladder structure.

To verify the results, an input file for the APLAC Simulation program was created, in which the elements of the ladder structure were described as being microwave transmission lines. All transmission lines will have the same length (defined by the cut-off frequency), but with impedance values defined by the different element values. The translation from ladder structure into microwave transmission lines circuit yields the realization as shown in Figure 3, and the i-input file for the APLAC program as listed in Figure 4. Since the intention is just to verify the calculations so far, the microwave circuit is also normalized to a cut-off frequency of a mere 1 Hz. In Figures 5 and 6, respectively the overall amplitude transfer characteristic (not showing repetition) of the distributed filter, and a zoomed in version of the passband are shown. Due to the non-linear transformation method used, together with our choice of normalizaion, we have to recalculate where the frequencies of interest ω_p and ω_s will result in now.

It is well known that the relation

$$f = \frac{\arctan(\omega)}{\frac{\pi}{4}}$$

exist, where f denotes the frequency along the horizontal axis in Figures 5 and 6, when ω is used along the axis of Figure 1 in our original design.

points of interest	ω	f
p_1	0.2833	0.3515
p_2	0.7508	0.8200
p_3	0.9746	0.9836
cut off	1.0000	1.0000
s_1	1.3481	1.1874
s_2	1.9480	1.3961
infinity	∞	2.0000

Measurements using the 'cursor' option in APLAC being in accordance with our calculations, prove the correctness of our complete design.

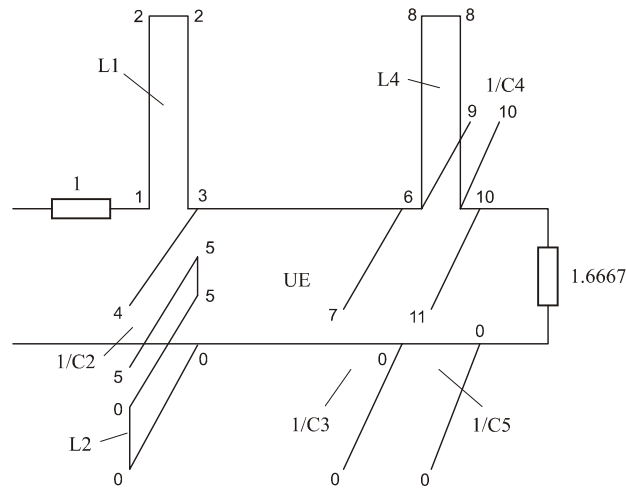


Figure 3: A realization in transmission lines of the ladder structure of Figure 2.

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Declare IVAR
+ L1 = 1.2749
+ C2 = 1.1062
+ L2 = 0.2382
+ UE = 2.7721
+ C3 = 1.1599
+ L4 = 1.3018
+ C4 = 0.4227
+ C5 = 0.5825

$-----
$ Transmission lines
$-----
Tline line1  1  3  2  2  EL_LENGTH 90 FC 2  Z  L1
Tline line2  3  5  4  5  EL_LENGTH 90 FC 2  Z  1/C2
Tline line3  5  0  0  0  EL_LENGTH 90 FC 2  Z  L2
Tline line4  3  0  6  0  EL_LENGTH 90 FC 2  Z  UE
Tline line5  6  0  7  0  EL_LENGTH 90 FC 2  Z  1/C3
Tline line6  6 10  8  8  EL_LENGTH 90 FC 2  Z  L4
Tline line7  6 10  9 10  EL_LENGTH 90 FC 2  Z  1/C4
Tline line8 10  0 11  0  EL_LENGTH 90 FC 2  Z  1/C5

$-----
$ 2-port definition
$-----
DefnPort ideal_splane_filt 2 1 0 1 10 0 1.6667

$-----
$ Output commands
$-----
Sweep "S-parameter Analyse"
+ LOOP 2000 FREQ LIN 0.0001 2
+ WINDOW 0
+ Y "|S21|" "(dB)" -70 0.0 GRID
+ WINDOW 1 + X "f" "Hz" 0.0001 1
+ Y "|S21|" "(dB)" -0.4 0.0 GRID

Show
+ WINDOW 0 Y MagdB(S(2,1))
+ WINDOW 1 Y MagdB(S(2,1))
EndSweep

```

Figure 4: Listing of the input file for APLAC.

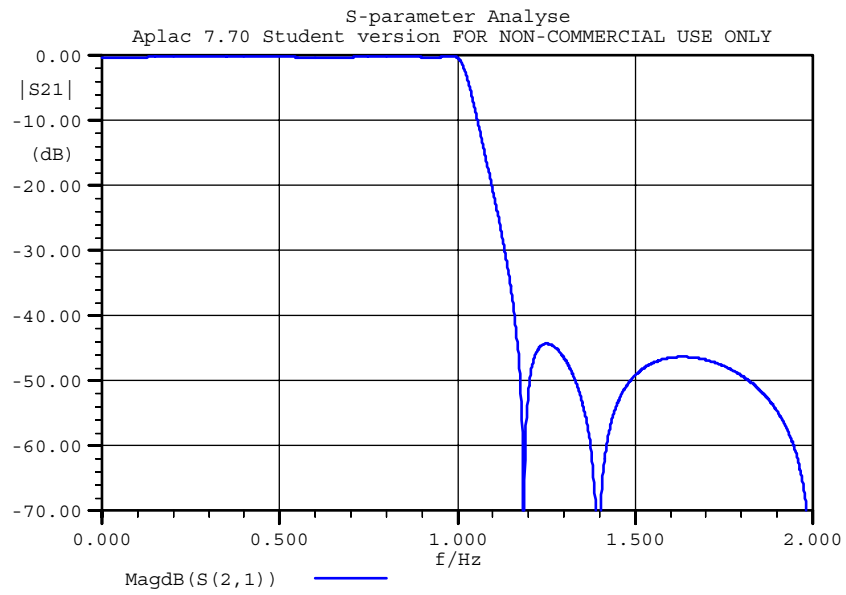


Figure 5: Output from the APLAC Simulator

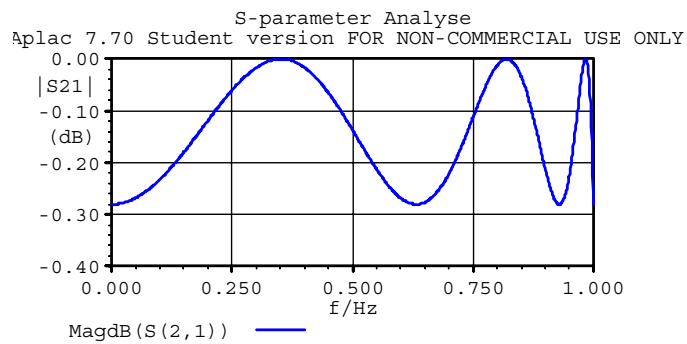


Figure 6: The passband, according to the APLAC Simulator.